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Aspects of Interacting Electromagnetic and Torsion Fields

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Abstract The interaction energy is studied for the coupling of axial torsion fields with photons in the presence of an external electromagnetic field. To this end, we compute the static quantum potential. Our discussion is carried out using the gauge-invariant but path-dependent variables formalism, which is alternative to the Wilson loop approach. Our results show that the static potential is a Yukawa correction to the usual static Coulomb potential. Interestingly, when this calculation is done by considering a mass term for the gauge field, the Coulombic piece disappears leading to a screening phase.

Keywords Gauge invariance · Screening · Static potential

1 Introduction

The formulation and possible experimental consequences of extensions of the Standard Model (SM), such as torsion fields, have been vastly investigated over the latest years [1–9]. As is well-known, this is because the SM does not include a quantum theory of gravitation. In fact, the necessity of a new scenario has been suggested to overcome difficulties theoretical in the quantum gravity research. In this respect we recall that string theories [10] provide a consistent framework to unify all fundamental interactions. We also point out that string theories are endowed with interesting features such as a metric, a scalar field (dilaton) and an antisymmetric tensor field of the third rank which is associated with torsion. It is worthy recalling at this stage that, in addition to the string interest, torsion fields have been discussed under a number of different aspects. For instance, in connection with the observed

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anisotropy of the cosmological electromagnetic propagation [11, 12], the dark energy problem [13], and in higher-dimensional theories [14, 15]. Also, new constraints on torsion from Lorentz violation have been studied in [16]. The ongoing activity of the Large Hadronic Collider (LHC) has also attracted interest in order to test the dynamical torsion parameters [17] and, related to this issue, the production of light gravitons [18–21] at accelerators justifies the study of dynamical aspects of torsion. It is worth recalling at this point that a connection between torsion and electromagnetic interaction has been pursued in an interesting line of investigation in [22, 23]. In these papers, the author shows that, if the vector and the axial-vector torsion components are present in the geometry of the space-time, then the torsion field equations yield an electromagnetic theory with magnetic monopoles, similar to the 2-photon electromagnetism formulated in [24] to describe magnetic monopoles.

With these considerations in mind, it should be interesting to acquire a better understanding what might be the observational signatures presented by both spin-1 and spin-0 states for the axial torsion field, S_μ, coupled to photons in the presence of an external background electromagnetic field. Hence, our purpose here is to investigate the impact of torsion on physical observables, in particular the static potential between two charges, using the gauge-invariant but path-dependent variables formalism, which is alternative to the Wilson loop approach [25–27]. Of special interest will be to explore the existence of duality (equivalence) between the familiar photon $U(1)_{QED}$ and a second massive gauge field living in the so-called hidden-sector $U(1)_h$ and torsion fields, by comparing physical quantities in both theories. As well as, the gauge-invariant but path-dependent variables formalism offers an alternative view in which some features of effective Abelian theories become more transparent.

2 Interaction Energy

We shall now discuss the interaction energy between static point-like sources for the model under consideration. To this end, we will compute the expectation value of the energy operator H in the physical state $|\Phi\rangle$ describing the sources, which we will denote by $\langle H\rangle_\Phi$. To carry out our study, we consider the Lagrangian density [1, 28]:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}S_{\mu\nu}^2 + \frac{1}{2}m^2 S_\mu^2 - \frac{b}{2}(\partial_\mu S^\mu)^2 + \frac{g}{4}S^\lambda \partial_\lambda \left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right), \tag{2.1}$$

where $S_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu$, $\widetilde{F}_{\mu\nu} \equiv 1/2\varepsilon_{\mu\nu\lambda\rho}F^{\lambda\rho}$, g is a coupling constant with dimension (-2) in mass units, and $b = \frac{m^2}{m_0^2}$. m and m_0, respectively, denote the masses of the spin-1 and spin-0 states for the torsion field (S_μ).

We further notice that the torsion tensor, $T_{\mu\nu\kappa}$, anti-symmetric in $\mu\nu$ can generally be split into three irreducible components:

- the tensor $R_{\mu\nu\kappa}$, such that $R^\mu_{\nu\mu} = 0$ and $\epsilon^{\lambda\mu\nu\kappa}R_{\mu\nu\kappa} = 0$, so that R presents 16 components,
- the trace $T^\mu = T^{\mu\nu}_\nu$,
- the (vector) pseudo-trace, $S_\mu = \epsilon_{\mu\nu\kappa\lambda}T^{\nu\kappa\lambda}$.

Moreover, the interaction of the torsion components $R_{\mu\nu\kappa}$ and T_μ with scalars and spinor is non-minimal; only S_μ can minimally couple to matter. So, from now on, we consider a dynamical model for the torsion in which only its axial part, S_μ, propagates.

According to the results of the paper of Ref. [8] on the constraints to be obeyed by quantum torsion, both the spin-1 and the spin-0 excitations of S_μ must be much more massive than the fundamental particles of the Standard Model. Also, if we assume that the spin-0 mode is much heavier than the spin-1 component of S_μ, the presence of the $(\partial_\mu S^\mu)^2$-term in (2.1) becomes harmless, in that the ghost mode that would run into troubles with unitarity is suppressed [28]. Then, considering the situation for which $m_0^2 \gg m^2 \gg m_{SM}^2$, where m_{SM}^2 stands for the masses of the fundamental particles of the Standard Model, we are allowed to integrate over the S_μ-field and consider an effective model that describes physics at scales $\ll m^2$.

Next, by integrating out the S_μ field in expression (2.1), and splitting $F_{\mu\nu}$ in the sum of a classical background, $\langle F_{\mu\nu}\rangle$, and a small fluctuation, $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, the corresponding Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4} f_{\mu\nu}^2 + \frac{g^2}{8}\left(v^{\mu\nu} f_{\mu\nu}\right)\frac{\Delta}{(b\Delta + m^2)}\left(v^{\lambda\gamma} f_{\lambda\gamma}\right), \tag{2.2}$$

where $\Delta \equiv \partial^\mu \partial_\mu$. Here, we have simplified our notation by setting $\varepsilon^{\mu\nu\alpha\beta}\langle F_{\mu\nu}\rangle \equiv v^{\alpha\beta}$. This effective theory thus provides us with a suitable starting point to study the interaction energy.

We now proceed to obtain the interaction energy in the $v^{0i} \neq 0$ and $v^{ij} = 0$ case (referred to as the magnetic one in what follows). In such a case, the Lagrangian density (2.2) reads as below:

$$\mathcal{L} = -\frac{1}{4} f_{\mu\nu}^2 + \frac{g^2}{8}\left(v^{0i} f_{0i}\right)\frac{\Delta}{(b\Delta + m^2)}\left(v^{0k} f_{0k}\right), \tag{2.3}$$

where $\mu, \nu = 0, 1, 2, 3$ and $i, k = 1, 2, 3$. To obtain the corresponding Hamiltonian, we must carry out the quantization of this theory. Before going into details, we call attention to the fact that the system described by (2.3) hides an important aspect of this effective theory, that is, it contains non-local time derivatives. We further recall that this paper is aimed at studying the static potential of the above theory, so that Δ can be replaced by $-\nabla^2$. However, at the moment for notational convenience we will maintain Δ, but it should be borne in mind that this paper essentially deals with the static case. Having made this observation, the canonical quantization of this theory from the Hamiltonian point of view follows straightforwardly. The Hamiltonian analysis starts with the computation of the canonical momenta $\Pi^\mu = f^{\mu 0} + \frac{g^2}{4} v^{0\mu}\frac{\Delta}{(b\Delta+m^2)} v^{0k} f_{0k}$. This yields the usual primary constraint $\Pi^0 = 0$, while the momenta are $\Pi_i = D_{ij} E_j$. Here $E_i \equiv f_{i0}$ and $D_{ij} = \delta_{ij} + \frac{g^2}{4} v_{i0}\frac{\Delta}{(b\Delta+m^2)} v_{j0}$. Since $\mathbf{D}$ is nonsingular, there exists its inverse, $\mathbf{D}^{-1}$. With this, the corresponding electric field can be written as $E_i = \frac{1}{\det D}\{\delta_{ij}\det D - \frac{g^2}{4} v_{i0}\frac{\Delta}{(b\Delta+m^2)} v_{j0}\}\Pi_j$. Therefore, the canonical Hamiltonian takes the form

$$H_C = \int d^3x \left\{-a_0 \partial_i \Pi^i + \frac{1}{2}\mathbf{B}^2\right\} - \int d^3x \frac{1}{2}\Pi_i \left[1 - \frac{g^2 \mathbf{v}^2}{4}\frac{\Delta}{(\xi^2\Delta + m^2)}\right]\Pi^i, \tag{2.4}$$

with $\xi^2 \equiv b + \frac{\mathbf{v}^2 g^2}{4} = b + \mathbf{v}^2 g^2 \mathcal{B}^2$. Here, $\mathbf{B}$ and $\mathcal{B}$ stand, respectively, for the fluctuating magnetic field and the classical background magnetic field around which the a^μ-field fluctuates. $\mathbf{B}$ is associated to the quantum a^μ-field: $B^i = -\frac{1}{2}\varepsilon_{ijk} f^{jk}$, whereas $\mathcal{B}_i$, according to our definition for the background $\langle F_{\mu\nu}\rangle$ in terms of $v_{\mu\nu}$ is given by $\mathcal{B}_i = \frac{1}{2} v_{0i}$. Temporal conservation of the primary constraint, Π_0, leads to the secondary constraint, $\Gamma_1(x) \equiv \partial_i \Pi^i = 0$. It is straightforward to check that there are no further constraints in the theory. Consequently, the extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^3x (c_0(x)\Pi_0(x) + c_1(x)\Gamma_1(x))$. Here $c_0(x)$ and $c_1(x)$ are arbitrary Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{a}_0(x) = [a_0(x), H] = c_0(x)$, which is an arbitrary function. Since $\Pi^0 = 0$ always, neither a^0 nor Π^0 are of interest in describing the system and may be discarded from the theory. As a result, the Hamiltonian becomes

$$H_C = \int d^3x \left\{ c(x) + \frac{1}{2}\mathbf{B}^2 \right\} - \int d^3x \frac{1}{2}\Pi_i \left[1 - \frac{g^2\mathbf{v}^2}{4}\frac{\Delta}{(\xi^2\Delta + m^2)} \right] \Pi^i, \tag{2.5}$$

where $c(x) = c_1(x) - a_0(x)$.

According to the usual procedure, we introduce a supplementary condition on the vector potential such that the full set of constraints becomes second class. A particularly convenient choice is found to be

$$\Gamma_2(x) \equiv \int_{C_{\xi x}} dz^\nu A_\nu(z) \equiv \int_0^1 d\lambda x^i A_i(\lambda x) = 0, \tag{2.6}$$

where λ $(0 \le \lambda \le 1)$ is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x - \xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice (2.6) leads to the Poincaré gauge [25, 26]. As a consequence, the only nontrivial Dirac bracket for the canonical variables is given by

$$\left\{ A_i(x), \Pi^j(y) \right\}^* = \delta_i^j \delta^{(3)}(x - y) - \partial_i^x \int_0^1 d\lambda x^j \delta^{(3)}(\lambda x - y). \tag{2.7}$$

We are now equipped to compute the interaction energy for the model under consideration. As mentioned before, in order to accomplish this purpose, we will calculate the expectation value of the energy operator H in the physical state $|\Phi\rangle$. Let us also mention here that, as was first established by Dirac [29], the physical state $|\Phi\rangle$ can be written as

$$|\Phi\rangle \equiv \left| \overline{\Psi}(\mathbf{y}) \Psi(\mathbf{y}\prime) \right\rangle = \overline{\psi}(\mathbf{y}) \exp\left(iq \int_{\mathbf{y}\prime}^{\mathbf{y}} dz^i A_i(z) \right) \psi(\mathbf{y}') \left|0\right\rangle, \tag{2.8}$$

where the line integral is along a spacelike path on a fixed time slice, and $|0\rangle$ is the physical vacuum state. Notice that the charged matter field together with the electromagnetic cloud (dressing) which surrounds it, is given by $\Psi(\mathbf{y}) = \exp(-iq \int_{C_{\xi\mathbf{y}}} dz^\mu A_\mu(z))\psi(\mathbf{y})$. Thanks to our path choice, this physical fermion then becomes $\Psi(\mathbf{y}) = \exp(-iq \int_{\mathbf{0}}^{\mathbf{y}} dz^i A_i(z))\psi(\mathbf{y})$. In other terms, each of the states ($|\Phi\rangle$) represents a fermion-antifermion pair surrounded by a cloud of gauge fields to maintain gauge invariance.

Taking into account the above Hamiltonian structure, we observe that

$$\Pi_i(x)|\overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\rangle = \overline{\Psi}(\mathbf{y})\Psi(\mathbf{y}')\Pi_i(x)|0\rangle + q\int_{\mathbf{y}}^{\mathbf{y}'} dz_i \delta^{(3)}(\mathbf{z} - \mathbf{x})|\Phi\rangle. \tag{2.9}$$

Having made this observation and since the fermions are taken to be infinitely massive (static) we can substitute Δ by $-\nabla^2$ in (2.5). Therefore, the expectation value $\langle H\rangle_\Phi$ is expressed as

$$\langle H\rangle_\Phi = \langle H\rangle_0 + \langle H\rangle_\Phi^{(1)} + \langle H\rangle_\Phi^{(2)}, \tag{2.10}$$

where $\langle H\rangle_0 = \langle 0|H|0\rangle$. The $\langle H\rangle_\Phi^{(1)}$ and $\langle H\rangle_\Phi^{(2)}$ terms are given by

$$\langle H\rangle_\Phi^{(1)} = -\frac{1}{2}\frac{bq^2}{(b+g^2\mathcal{B}^2)}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i'\delta^{(3)}(\mathbf{x}-\mathbf{z}')\left(1-\frac{\nabla^2}{M^2}\right)_x^{-1}\int_{\mathbf{y}}^{\mathbf{y}'} dz^i\delta^{(3)}(\mathbf{x}-\mathbf{z}), \quad (2.11)$$

and

$$\langle H\rangle_\Phi^{(2)} = \frac{M^2q^2}{2}\int d^3x \int_{\mathbf{y}}^{\mathbf{y}'} dz_i'\delta^{(3)}(\mathbf{x}-\mathbf{z}')\left(\nabla^2-M^2\right)_x^{-1}\int_{\mathbf{y}}^{\mathbf{y}'} dz^i\delta^{(3)}(\mathbf{x}-\mathbf{z}), \quad (2.12)$$

where $\mathcal{M}^2 \equiv \frac{m^2}{\lambda^2} = \frac{m^2}{b+g^2\mathcal{B}^2}$. Following our earlier procedure [27, 30, 31], we see that the potential for two opposite charges, located at $\mathbf{y}$ and $\mathbf{y}'$, takes the form

$$V = -\frac{q^2}{4\pi}\left[\frac{1}{L}-\left(1-\frac{b}{\lambda^2}\right)\frac{e^{-\mathcal{M}L}}{L}\right], \quad (2.13)$$

where $|\mathbf{y}-\mathbf{y}'| \equiv L$. Hence we see that the static potential profile is a Yukawa correction to the usual Coulomb potential. Notice that expression (2.13) is spherically symmetric, although the external fields break the isotropy of the problem in a manifest way. The Yukawa-type component of the potential above vanishes whenever $b=\xi^2$, in other terms, when the external magnetic field is switch off.

Before going ahead, we would like to note that from Lagrangian (2.1) the torsion-electromagnetic field interaction term can be also written as $\mathcal{L}_{int} = -g(\partial_\lambda S^\lambda)\mathbf{E}\cdot\mathbf{B}$, up to a surface term. By splitting the classical background and the quantum fluctuation of the electromagnetic field, $\mathbf{E} = \langle\mathbf{E}\rangle + \mathbf{e}$, and $\mathbf{B} = \langle\mathbf{B}\rangle + \mathbf{b}$, we readily see that in the case the background is purely magnetic, $\langle\mathbf{E}\rangle = 0$, torsion couples to the fluctuation $\mathbf{e}$ via $\langle\mathbf{B}\rangle$. Since we are seeking the interparticle potential in the static regime, the interaction term with $\langle\mathbf{B}\rangle$ present is $-g(\partial_\lambda S^\lambda)\langle\mathbf{B}\rangle\cdot\mathbf{e} = -g(\partial_\lambda S^\lambda)\langle\mathbf{B}\rangle\cdot\nabla\varphi$, so that, we are lead to conclude that it is the interchange of the scalar φ the responsible for the Yukawa-like piece of the potential.

Within this framework, the next step is to extend further the previous analysis by considering the effect of a mass term for the A_μ field, which may arise via a Higgs or Stüeckelberg mechanism. In addition, similar results have been obtained in the context of electrodynamics in the presence of magnetic monopoles [32, 33]. For this purpose, we begin by writing down the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}\overline{m}^2A_\mu^2 - \frac{1}{4}S_{\mu\nu}^2 + \frac{1}{2}m^2S_\mu^2 - \frac{b}{2}\left(\partial_\mu S^\mu\right)^2 + \frac{g}{4}S^\lambda\partial_\lambda\left(F_{\mu\nu}\widetilde{F}^{\mu\nu}\right). \quad (2.14)$$

It is straightforward to see that gauge invariance has been broken in (2.14) and one could argue about the possibility of getting a gauge invariant result for the static potential between test charges from (2.14). There are at least two available options to solve this apparent inconsistency. As is well known, one way is to restore gauge invariance by inserting Stüeckelberg compensating fields into (2.14). Once the compensators are integrated out the resulting model is explicitly gauge invariant. In alternative we can follow the Hamiltonian formulation along the lines of [25–27]. As a consequence, the new effective Lagrangian density takes the form

$$\mathcal{L}_{eff} = -\frac{1}{4}f_{\mu\nu}\left(1+\frac{\overline{m}^2}{\Delta}\right)f^{\mu\nu} + \frac{g^2}{8}\left(v^{\mu\nu}f_{\mu\nu}\right)\frac{\Delta}{(b\Delta+m^2)}\left(v^{\lambda\gamma}f_{\lambda\gamma}\right). \quad (2.15)$$

Next, after splitting $F_{\mu\nu}$ in the sum of a classical background $\langle F_{\mu\nu}\rangle$ and a small fluctuation $f_{\mu\nu}$, the corresponding Lagrangian density becomes

$$\mathcal{L}_{eff} = -\frac{1}{4} f_{\mu\nu}\left(1+\frac{\overline{m}^2}{\Delta}\right) f^{\mu\nu} + \frac{g^2}{8}\left(v^{0i} f_{0i}\right)\frac{\Delta}{(b\Delta+m^2)}\left(v^{0k} f_{0k}\right). \tag{2.16}$$

To get the last equation we have considered the magnetic case ($v^{0i} \neq 0$ and $v^{ij}=0$).

Again, following the same steps that lead to (2.13), we see that the potential for two opposite charges, located at $\mathbf{y}$ and $\mathbf{y}'$, takes the form

$$V = -\frac{q^2}{4\pi}\frac{1}{(b+g^2\mathcal{B}^2)}\frac{1}{\sqrt{\xi^4-4M^4}}\left[\left(M_1^2 b - m^2\right)\frac{e^{-M_1 L}}{L} - \left(M_2^2 b - m^2\right)\frac{e^{-M_2 L}}{L}\right], \tag{2.17}$$

where $M^4 = \frac{m^2\overline{m}^2}{(b+g^2\mathcal{B}^2)}$, $\xi^2 = \frac{m^2}{(b+g^2\mathcal{B}^2)}$, $M_1^2 = \frac{1}{2}(\alpha^2+\sqrt{\alpha^4-4\beta^4})$ and $M_2^2 = \frac{1}{2}(\alpha^2 - \sqrt{\alpha^4-4\beta^4})$, with $\alpha^2 \equiv \frac{m^2+\overline{m}^2}{(b+g^2v^2/4)}$ and $\beta^2 \equiv \sqrt{\frac{m^2\overline{m}^2}{(b+g^2v^2/4)}}$. Hence we see that for $\overline{m}=0$, expression (2.17) reduces to expression (2.13). Therefore, when the analysis is done with a mass term for the A_μ field, the surprising result is that the theory describes an exactly screening phase.

3 Final Remarks

In summary, by using the gauge-invariant but path-dependent formalism, we have computed the static potential for a theory which includes both spin-1 and spin-0 states for the axial torsion field S_μ coupled to photons, in the case when there are nontrivial constant expectation values for the gauge field strength $F_{\mu\nu}$.

Our analysis reveals that the static potential profile obtained from both the coupling between the familiar massless electromagnetism $U(1)_{QED}$ and a hidden-sector $U(1)_h$ and torsion fields are analogous. This means that the physical content for both theories is equivalent. The above connections are of interest from the point of view of providing unifications among diverse models as well as exploiting the equivalence in explicit calculations, as we have shown.

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